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Probabilistic models considering dependent relation in reasoning for decision-making

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Abstract

This paper presents the probabilistic models considering dependent relation for solving the reasoning problem in decision-maiking. It is important for constructing the joint probability-distribution to consider the dependence of events. In this paper, we classify the dependent relations. Moreover, we apply the fuzzy probability to calculation of the dependence-index. Some vagueness is included in the dependency. Also, we define the fuzzy dependence-index to consider dependency with fuzziness. Using the fuzzy dependence-index, we calculate the joint probability of multi-events and construct the probabilistic model.

1 Introduction

Study of reasoning algorithm in decision-making has been widely appeared in practical application like diagnosis or prediction problem under uncertainty. Concerning the fact, research about how to build the probabilistic model in statistical study has become important. Probabilistic models are constructed from the joint probability-distribution of each event based on the existing data[1]. There are various types of dependency for causal relationship and correlation among events, and it is significant to argue how to handle these relations.

In this paper, we consider about the probabilistic model taking dependent relation into account. We introduce the product operator which is useful to calculate the joint probability. We first classify dependent relations. Then we introduce the dependence index for each relation.

In the existing research, the correlation of random variables or the conditional probability rules is used for detecting dependencies among events. However, it is difficult to define the index using definite value, because the dependence varies delicately with the situation where events occur. In this case, it may be desirable to be expressed with fuzzy numbers from the experience or the knowledge of experts.

As in the uncertainty of events, we assume dependent relations include a certain kind of vagueness. It is dealt as fuzziness in the paper. The dependency among events will also become uncertain when human's subjective judgment is included in the events on the sample space. We define the fuzzy dependence-index to treat conditional probability as a kind of fuzzy probability. It becomes easy to build the probabilistic model of multi-events with some dependency by introducing the product operation based on the fuzzy dependence-index.

2 Dependent relation for joint probability

2.1 Joint probability

In this section, we explain the theorem of joint probability. The joint probability of the event E_1 and E_2 is expressed by using conditional probability $P(E_2|E_1)$ in multiplication rule for probability as

$$P(E_1 \cap E_2) = P(E_1)P(E_2|E_1) \quad (1)$$

If there are some dependencies between E_1 and E_2, there exists $P(E_2|E_1) \neq P(E_2)$. The degree of dependency is given by dependence-index. Mabuchi[2, 3] defined the joint probability with dependency using dependence-index and fuzzy set product operator. The definition of fuzzy set operation with the dependence-index is as follows:

$$P(E_1 \cap E_2) = \gamma_{E_1E_2}P(E_1)P(E_2) \quad (2)$$

where

$$\gamma_{E_1E_2} = \frac{P(E_2|E_1)}{P(E_2)} \quad (3)$$

and $\gamma_{E_1E_2}$ is called as the dependence-index of E_1 and E_2. Equation(2) follows by the general definition of probability theory. The parameter of dependence relies on the conditional probability among events. We classify dependent relations based on equation(3). In calculating joint probabilities, we utilize equation(2) and then construct the probabilistic model.

3 Dependent relation with fuzziness

First, we classify the dependent relation into five parts. Then, we define the fuzzy dependence-index of equation(3) based on the classification.

The dependence-index can be computed by examining the correlation of each event in a certain sample space[4, 5]. As another way, the dependence-index is derived by the conditional probability among events. However, it is difficult to define the index as a definite value because the dependency of events includes human's subjective vagueness. For example, when we think about the relation between a pneumonia and a cough, a cough does not necessarily mean getting a pneumonia. The factors such as environment or other events may influence that relation. It is difficult for us to define the index as the definite values because the dependent relation changes along the environment where each event occurs and is observed.Then we should consider that the index includes fuzziness. In this case, we should express the index of dependent relation with not definite values but fuzzy numbers.

3.1 Classification of dependent relations

We classify kinds of dependent relations through equation(3) as follows:

i. Independence

ii. Perfect dependence

iii. Positive partial dependence

iv. Negative partial dependence

v. Opposite

The relation is described as the positive dependence in which the joint probability is higher than that of independence. The extreme case of that is the perfect dependence. On the other hand, negative dependence is a relation contrary to positive dependence. In this case, the degree of joint probability is lower than independent case.

i) Independence

When the probability of each event can not be influenced by other events, we have $P(E_i|E_j) = P(E_i)$. This dependent relation is called "Independence" of which γ of multi events becomes

$$\gamma = 1 \tag{4}$$

ii) Perfect dependence

We call the relation in which one event is completely contained in another as "Perfect dependence". $P(E_i \mid E_j) = 1$ is obtained when $E_j \in E_i$. When this value of the conditional probability is applied to equation(3), $\gamma_{E_iE_j}P(E_i) = 1$ is obtained. Then, the dependence-index is derived as

$$\gamma = min\left(\frac{1}{e_1}, \frac{1}{e_2}, \dots \frac{1}{e_n}\right) \tag{5}$$

where $e_i = P(E_i)$.

iii) Positive partial dependence

If $P(E_i|E_j) > P(E_i)$, we call the relation as "Positive partial dependence". In this case, we have $\gamma_{E_iE_j}P(E_i) > P(E_i)$. Further, the conditional probability is $P(E_i|E_j) < 1$ due to partial dependence, then the dependence-index γ is acquired in the range of

$$1 < \gamma < min\left(\frac{1}{e_1}, \frac{1}{e_2}, \dots \frac{1}{e_n}\right) \tag{6}$$

iv) Negative partial dependence

We called the case as "Negative partial dependence" when the independent probability is lower than the conditional probability due to the influence on each other. When $P(E_i|E_j) < P(E_i)$, we have $\gamma_{E_iE_j}P(E_i) < P(E_i)$. The dependence-index is given as

$$0 < \gamma < 1 \tag{7}$$

v) Opposite

When $P(E_i \cap E_j) = \phi$, the dependent relation of the events is exclusive. In the occasion, the joint probability is 0. Then γ is

$$\gamma = 0 \tag{8}$$

3.2 Fuzzy dependence-index

In this paper, we define the fuzzy dependence-index by fuzzy probability[6, 7] applied to the probability for fuzzification of the dependence-index. The fuzzy probability, here, is different from the probability of fuzzy event. This means the fuzzy set based on probability measure as

$$\mu_P : P \rightarrow [0,1] \tag{9}$$

The concept of fuzzy probability realizes the fuzzification of the dependence-index by quantitatively expressing as the probability. In this research, the probability measure P of "the probability is roughly m" is expressed as triangler membership function of

$$\mu_P(x) = max\left(0, 1 - \frac{|x-m|}{d}\right), d > 0 \tag{10}$$

Where m is a modal value of triangler fuzzy numbers. We express the degree of probability as fuzzy probability with the membership function when we get "about 0.7". This is because the fuzzy number 0.7 involving 0.6 and 0.8 is more suitable to the human's language perception than the definite value 0.7.

We extend $P(A)$ as the fuzzy number P_A by fuzzification of the probability. The fuzzy set of P_{A_i} is illustrated as

$$P_{A_i} = \int_{a_i-d}^{a_i+d} 1 - \frac{|v-a_i|}{d} / v \tag{11}$$

with the triangler membership function, where $a_i = P(A_i)$, $i = 1, 2, \ldots, n$ and $v = [0,1]$ is the degree of probability. We define the fuzzy dependence-index by the fuzzy probability based on equation(11).

Meanwhile, in the case of a continuous membership function, the result of joint probability is also expressed as a function. When one want to express the dependence-index without the function, it is possible to consider the interval instead of the fuzzy numbers. Then, we also define the dependence index with the interval obtained from α-level cut. According to equation(11), it is expressed with the closed interval depending on α that gives the α-level cut $(P_{A_i})_\alpha$. Then, $(P_{A_i})_\alpha$ is given as

$$(P_{A_i})_\alpha = \{v \in R^1 \mid \mu_{P_{A_i}}(v) \geq \alpha\} = [p^L_{A_{i(\alpha)}}, p^R_{A_{i(\alpha)}}] \tag{12}$$

We give the following definitions of the fuzzy dependence-index $\tilde{\gamma}$ with the fuzzy probability and the dependence index with α-level cut.

i) Independence

In this case, the fuzzy set Γ_I of the fuzzy dependence-index $\tilde{\gamma}$ is defined as

$$supp(\Gamma_I) = \left\{\tilde{\gamma} \mid \frac{1-d}{1+d} \leq \tilde{\gamma} \leq \frac{1+d}{1-d}\right\} \tag{13}$$

$$\Gamma_I = \begin{cases} \int_1^{\frac{1+d}{1-d}} \left(1 - \frac{(1-d)(\tilde{\gamma}-1)}{2d}\right) / \tilde{\gamma} & if \quad 1 < \tilde{\gamma} \leq \frac{1+d}{1-d} \\ 1/\tilde{\gamma} & if \quad \tilde{\gamma} = 1 \\ \int_{\frac{1-d}{1+d}}^{1} \left(1 - \frac{(1+d)(1-\tilde{\gamma})}{2d}\right) / \tilde{\gamma} & if \quad \frac{1-d}{1+d} \leq \tilde{\gamma} < 1 \end{cases} \tag{14}$$

As α-level cut, we have

$$p^L_{1_{(\alpha)}} \leq \gamma \leq p^R_{1_{(\alpha)}} \tag{15}$$

where $p_{1_{(\alpha)}}$ is α-level cut of the fuzzy number 1.

ii) Perfect dependence

In the perfect dependence, the fuzzy set Γ_{PD} of $\tilde{\gamma}$ is defined as

$$supp(\Gamma_{PD}) = \left\{\tilde{\gamma} \mid \frac{1}{a_m+d} \leq \tilde{\gamma} \leq \frac{1}{a_m-d}\right\} \tag{16}$$

$$\Gamma_{PD} = \begin{cases} \int \left(1 - \frac{a_m(a_m+d)(\tilde{\gamma}-a_m^{-1})}{d}\right) / \tilde{\gamma} & if \quad \frac{1}{a_m} < \tilde{\gamma} \leq \frac{1}{a_m-d} \\ 1/\tilde{\gamma} & if \quad \tilde{\gamma} = \frac{1}{a_m} \\ \int \left(1 - \frac{a_m(a_m+d)(a_m^{-1}-\tilde{\gamma})}{d}\right) / \tilde{\gamma} & if \quad \frac{1}{a_m+d} \leq \tilde{\gamma} < \frac{1}{a_m} \end{cases} \tag{17}$$

where

$$a_m = \max_{1 \leq i \leq n} [a_i] \quad , \quad d \leq a_m \tag{18}$$

In the case of α - level cut, according to equation(12), γ must lie within

$$\gamma = \left[\frac{1}{\max_{1 \leq i \leq n} [p^R_{A_{i(\alpha)}}]}, \frac{1}{\max_{1 \leq i \leq n} [p^L_{A_{i(\alpha)}}]}\right] \tag{19}$$

iii) Positive partial dependence

In the case of the positive partial dependence, the fuzzy set of $\tilde{\gamma}$ is expressed as Γ_{PPD} and defined as

$$supp(\Gamma_{PPD}) = \left\{\tilde{\gamma} \mid \frac{1-d}{1+d} < \tilde{\gamma} < \frac{1}{a_m-d}\right\} \tag{20}$$

$$\Gamma_{PPD} = \begin{cases} \int \left(1 - \frac{v_p(v_p+d)(\tilde{\gamma} - v_p^{-1})}{d}\right)/\tilde{\gamma} & if \quad \frac{1}{v_p} < \tilde{\gamma} \leq \frac{1}{v_p - d} \\ 1/\tilde{\gamma} & if \quad \tilde{\gamma} = \frac{1}{v_p} \\ \int \left(1 - \frac{v_p(v_p+d)(v_p^{-1} - \tilde{\gamma})}{d}\right)/\tilde{\gamma} & if \quad \frac{1}{v_p + d} \leq \tilde{\gamma} < \frac{1}{v_p} \\ \int 0/\tilde{\gamma} & if \quad \frac{1}{v_p - d} < \tilde{\gamma} < \frac{1}{a_m - d} \\ & or \quad \frac{1-d}{1+d} < \tilde{\gamma} < \frac{1}{v_p + d} \end{cases} \tag{21}$$

where $a_m < v_p < 1$, $d \leq v_p$, $v_p = \gamma^{-1}$.

As α-level cut, we have

$$p_{1(\alpha)}^L < \gamma < \frac{1}{\max\limits_{1 \leq i \leq n} [p_{A_{i(\alpha)}}^L]} \tag{22}$$

iv) Negative partial dependence

The fuzzy set of $\tilde{\gamma}$ is expressed as Γ_{NPD} in this case. Γ_{NPD} is defined as

$$supp(\Gamma_{NPD}) = \left\{ \tilde{\gamma} \mid 0 < \tilde{\gamma} < \frac{1+d}{1-d} \right\} \tag{23}$$

$$\Gamma_{NPD} = \begin{cases} \int \left(1 - \frac{v_n(v_n+d)(\tilde{\gamma} - v_n^{-1})}{d}\right)/\tilde{\gamma} & if \quad \frac{1}{v_n} < \tilde{\gamma} \leq \frac{1}{v_n - d} \\ 1/\tilde{\gamma} & if \quad \tilde{\gamma} = \frac{1}{v_n} \\ \int \left(1 - \frac{v_n(v_n+d)(v_n^{-1} - \tilde{\gamma})}{d}\right)/\tilde{\gamma} & if \quad \frac{1}{v_n + d} \leq \tilde{\gamma} < \frac{1}{v_n} \\ \int 0/\tilde{\gamma} & if \quad \frac{1}{v_n - d} < \tilde{\gamma} < \frac{1+d}{1-d} \\ & or \quad 0 < \tilde{\gamma} < \frac{1}{v_n + d} \end{cases} \tag{24}$$

where $1 < v_n < \infty$, $d \leq v_n$, $v_n = \gamma{-}1$.

As α-level cut, we have

$$0 < \gamma < p_{1(\alpha)}^R \tag{25}$$

v) Opposite

In the opposite case. the fuzzy set Γ_O is defined as

$$supp(\Gamma_O) = \left\{ \tilde{\gamma} \mid \tilde{\gamma} = 0 \right\} \tag{26}$$

Then, Γ_O can be estimated as follows.

$$\Gamma_O = 0 \tag{27}$$

4 Conclusion

This paper has presented the fuzzy dependence-index using the fuzzy probability. Applying the method that the dependence-index could be computed from the probability of events, the fuzzy index is introduced from fuzzification of the probabilities. We have defined the fuzzy dependence-index based on five classifications. This approach enables us to construct the probabilistic models considering the fuzziness among the relation of events.

Moreover, we defined the dependence-index as the interval with α-level cut. Then it is possible to compute the joint probability without the functions.

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